Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced Subsidiary

Mock Paper

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae	Answer Booklet

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 4 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Printer's Log. No.
N33687A

N 3 3 6 8 7 A

W850/XXXX/57570 2/2

Turn over

edexcel
advancing learning, changing lives

1.

$$\mathbf{R} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \mathbf{S} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

(a) Find $\mathbf{R}^2$.

(1)

(b) Find **RS**.

(2)

(c) Describe the geometrical transformation represented by **RS**.

(2)

(Total 5 marks)

---

2. A point P with coordinates (x, y) moves so that its distance from the point $(–3, 0)$ is equal to its distance from the line $x = 3$.

Find a cartesian equation for the locus of P.

(Total 3 marks)

3.

$$z = 1 + \mathrm{i}\sqrt{3}$$

Express in the form $a + \mathrm{i}b$, where a and b are real.

(a) $z^2 + z$,

(3)

(b) $\dfrac{z}{3 - z}$,

giving the exact values of a and b in each part.

(3)

(Total 6 marks)

4.

$$\mathrm{f}(x) = x^3 - 4x^2 + 5x - 3$$

The equation $\mathrm{f}(x) = 0$ has a root α in the interval $(2, 3)$.

(a) Use linear interpolation on the end points of this interval to obtain an approximation for α.

(4)

(b) Taking 2.5 as a first approximation to α, apply the Newton - Raphson procedure once to $\mathrm{f}(x)$ to obtain a second approximation to α. Give your answer to 2 decimal places.

(5)

(Total 9 marks)

---

5. Given that a and b are non-zero constants and that

$$\mathbf{X} = \begin{pmatrix} a & 2b \\ -a & 3b \end{pmatrix},$$

(a) find $\mathbf{X}^{-1}$, giving your answer in terms of a and b.

(3)

Given also that $\mathbf{Z}\,\mathbf{X} = \mathbf{Y}$, where $\mathbf{Y} = \begin{pmatrix} 3a & b \\ a & 2b \end{pmatrix}$,

(b) find $\mathbf{Z}$, simplifying your answer.

(4)

(Total 7 marks)

---

6. (a) Use the standard results for $\sum_{r=1}^{n} r$ and for $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r(2r^2 - 6) = \frac{1}{2}n(n+1)(n+3)(n-2).$$

(4)

(b) Hence calculate the value of $\sum_{r=10}^{50} r(2r^2 - 6)$.

(2)

(Total 6 marks)

---

7. The quadratic equation

$$z^2 + 10z + 169 = 0$$

has complex roots z_1 and z_2.

(a) Find each of these roots in the form $a + b\mathrm{i}$.

(3)

(b) Find the modulus and argument of z_1 and of z_2.
Give the arguments in radians to 3 significant figures.

(5)

(c) Illustrate the two roots on a single Argand diagram.

(2)

(d) Find the value of $|z_1 - z_2|$.

(2)

(Total 12 marks)

---

8. The rectangular hyperbola H has equation $xy = c^2$.

The point $(3t, \frac{3}{t})$ is a general point on this hyperbola.

(a) Find the value of c^2.

(1)

(b) Show that an equation of the normal to H at the point $(3t, \frac{3}{t})$ is

$$y = t^2 x + (\frac{3}{t} - 3t^3)$$

(5)

The point P on H has coordinates (6, 1.5). The tangent to H at P meets the curve again at the point Q.

(c) Find the coordinates of the point Q.

(7)

(Total 13 marks)

---

9. (a) A sequence of numbers is defined by

$$u_1 = 3 \text{ and } u_{n+1} = 3u_n + 4 \text{ for } n \geqslant 1.$$

Prove by induction that

$$u_n = 3^n + 2(3^{n-1} - 1) \text{ for } n \in \mathbb{Z}^+.$$

(5)

(b) $$\mathbf{A} = \begin{pmatrix} 4 & 0 \\ 9 & 1 \end{pmatrix}.$$

(i) Prove by induction that

$$\mathbf{A}^n = \begin{pmatrix} 4^n & 0 \\ 3(4^n - 1) & 1 \end{pmatrix} \text{ for } n \in \mathbb{Z}^+.$$

(7)

(ii) Determine whether the result $\mathbf{A}^n = \begin{pmatrix} 4^n & 0 \\ 3(4^n - 1) & 1 \end{pmatrix}$

is also valid for $n = -1$.

(2)

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END